COINCIDENCE STUDIES OF THE REACTIONS
$^3$He + $^2$H AND $^4$He + $^2$H

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Abstract: The reactions $^2$H($^4$He, $^4$He p)n, $^2$H($^3$He, $^3$He p)n and $^2$H($^3$He, $^3$H p)p were studied at incident helium energies of 18.0 and 24.0 MeV. In the $^2$H($^4$He, $^4$He p)n reaction the $^4$He and p were detected in coincidence at angles which allow low relative momentum of the proton and neutron in the final state. The singlet p-n final state interaction (FSI) was not observed (limit < 3% of the triplet FSI), in agreement with the prediction of isospin conservation. In the $^2$H($^3$He, $^3$He p)n reaction the singlet p-n FSI was observed. Analysis of these data for the extraction of the p-n scattering length was performed. In the same reaction, a measurement at quasi-free (neutron spectator) angles showed that the spectator-pole cross-section peak was shifted in energy from the value predicted by the simple knock-out model. This effect was reproduced by adding to the knock-out calculation "Coulomb pushing" between the incident helium nucleus and deuteron. The first and second excited states of $^4$He were observed at excitation energies ($E^*$) and widths ($\Gamma$) of $E^* = 20.014 \pm 0.02$ MeV, $\Gamma = 0.124 \pm 0.02$ MeV and $E^* = 21.364 \pm 0.10$ MeV, $\Gamma = 2.04 \pm 0.2$ MeV, respectively. Contributions probably due to higher $^4$He states were seen in the data. Angular correlation results were compatible with a spin zero assignment to the first excited state of $^4$He.

Nuclear Reactions $^2$H($^4$He, $^4$He p)n, $^2$H($^3$He, $^3$He p)n, $^2$H($^3$He, $^3$H p)p, $E = 18.0$, $24.0$ MeV; measured $a(E; E_x, E_y, \theta_x, \theta_y)$. $^4$He deduced levels, J. Enriched target.

1. Introduction

There is only one three-body channel open to reactions of the system $^4$He + $^2$H at incident energies used in this work, namely

$$^4$$He + $^2$H $\rightarrow$ $^4$He + p + n, \hspace{1cm} Q = -2.226$ MeV.  \hspace{1cm} (1)

Apart from the direct three-body break-up, the following reaction mechanisms are possible: (i) proton-neutron final state interaction (FSI), (ii) sequential decay through states of $^5$He, (iii) sequential decay through states of $^5$Li and the experimental variables may be selected in such a way as to concentrate on each mechanism separately.

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An interesting feature of reaction (1) is that the p-n FSI in the singlet state is forbidden by isospin conservation. Thus at regions of the coincidence energy spectra characterized by low p-n relative energy, the nonresonant triplet interaction is expected to dominate. One purpose of the work reported here is to search for a possible admixture of p-n singlet FSI in the reaction (1).

The reaction \(^3\text{He} + ^2\text{H}\) produces the following three-particle final states:

\[
\begin{align*}
^3\text{He} + ^2\text{H} &\rightarrow ^3\text{He} + p + n, & Q &= -2.226 \text{ MeV}, \\
^3\text{He} + ^2\text{H} &\rightarrow ^3\text{H} + p + p, & Q &= -1.461 \text{ MeV}, \\
^3\text{He} + ^2\text{H} &\rightarrow ^2\text{H} + ^2\text{H} + p, & Q &= -5.493 \text{ MeV}.
\end{align*}
\]

Among the phenomena we may expect to observe are:

(i) proton-neutron FSI, (ii) proton-proton FSI, (iii) sequential decay through states of \(^4\text{He}\), (iv) sequential decay through states of \(^4\text{Li}\).

In the present experiment the angles of observation were chosen in such a way as to enhance processes (i) and (iii). The p-n FSI in reaction (2a) is allowed in both spin states. Reaction (2b) contains two identical particles in the final state, and the matrix elements for the interaction between \(^3\text{H}\) and either proton are expected to add coherently.

Experiments on the reaction (2) are summarized in refs. \(^1,2\)). Most measure the spectrum of one of the particles in the final state. Coincident charged-particle spectra have been obtained with either deuterons \(^3\)) or \(^3\text{He}\) as projectiles \(^4-6\)). Most of the above processes have been previously observed in coincident spectra and the energy and width of the first two excited states of \(^4\text{He}\) have been extracted \(^6\)). The present work attempts to investigate further and with higher resolution processes (i) and (iii) and extract with greater precision the energy and width of the first two excited states of \(^4\text{He}\). Angular correlations of the decay particles from the first excited state of \(^4\text{He}\) were also measured.

2. Experimental procedure

2.1. APPARATUS

\(^3\text{He}^-\) and \(^4\text{He}^-\) beams were produced by an ion source \(^7\)) which uses Li vapor charge exchange and were accelerated by the Rutgers-Bell FN tandem Van de Graaff accelerator. Typical target currents obtained were 10-30 nA. Deuterated polyethylene (CD\(_2\)) targets of 0.5-0.6 mg/cm\(^2\) were used.

The scattering chamber and detection apparatus are shown in fig. 1. This represents a small modification of the apparatus employed in ref. \(^8\)). Two ORTEC 1 mm thick surface barrier detectors were used for all measurements. The detector temperature was lowered to approximately \(-40^\circ\text{C}\) by cooling with a recirculating freon system. The detectors were placed at a distance of 8.6 cm from the target with detector collimators 0.3 cm wide and 1.0 cm high. This corresponds to an angular acceptance of
± 1° in the reaction plane. The ratio ½ of width to height in the rectangular collimators was compatible with the value of ∂E/∂Φ and ∂E/∂Θ, the energy shift of the kinematic line due to the polar angles Φ and Θ; the value of (∂E/∂Φ)/(∂E/∂Θ) near the reaction plane is approximately ½. The solid angle subtended by each detector in the above configuration is ΔΩ = 4.06 × 10⁻³ sr.

As shown in fig. 1 a fast "time" pulse was derived from each detector by a very high input impedance fast preamplifier ⁹). The slower "energy" (E) signal was amplified by a charge-sensitive preamplifier ¹⁰) employing field-effect transistors. The preamplified signals were conveyed to the control room from the target room through a signal-handling system ¹¹) where they were further amplified by means of two TC-200 Tennelec amplifiers. These signals were then fed into two 50 MHz 4096-channel analog-to-digital converters ¹²) (ADC's) provided with gated DC restorers. Each ADC was stabilized with two precision pulses which were fed into the preamplifiers through the detector bias cables. By traveling through the same electronic arrangement, the servo-pulses were thus made to simulate detector signals. The ADC rundown slope and baseline intercept was then varied to force the higher pulse into the top channel of the ADC scale and the lower pulse into the channel at ½ full scale (channels 4096 and 1024, respectively for the 4096-channel scale). In this way both gain and zero-intercept drifts in the electronics were continuously checked and compensated. The two signals from the fast preamplifiers triggered two E. G. & G.
Model T101/N discriminators which provided the START and STOP pulses for a E.G. & G. Time-to-Amplitude Converter (TAC). The TAC output, in a range selected by an E.G. & G. Model AN109 Biased Amplifier, provided the input of a third "TIME" ADC. This timing information was used to effect particle identification as explained in the next section. At the same time the biased amplifier output gated the two "E" (Energy) ADC's.

Data from the three ADC's were stored in an on-line SDS-910 computer. In the coincidence operation mode the computer is programmed as a $64 \times 64 \times 6$ analyzer. The main program of the three-dimensional analyzer, called SARDINE, displays the computer memory contents on a cathode-ray oscilloscope screen in six $64 \times 64$ rectangular channel grids with the vertical deflection proportional to the number of events in each channel.

2.2. KINEMATICS AND PARTICLE IDENTIFICATION PROCEDURE

The kinematics of three-body reactions is given in detail elsewhere \textsuperscript{8,14,15}. We consider reactions of the form

$$1 + 2 \rightarrow 3 + 4 + 5,$$

where, in our convention, particles 3 and 4 are detected at fixed angles. A complete measurement of the final state in such a reaction involves the specification of nine scalar parameters, which are readily reduced to five by applying energy and momentum conservation. In the present experiment seven scalar parameters are measured, namely the incident beam energy $T_1$, the two outgoing particle energies $T_3$ and $T_4$, and the relevant emission angles of these particles $\Theta_3$, $\Theta_4$, $\Phi_3$ and $\Phi_4$. This amounts to the measurement of six independent scalar variables, since, due to cylindrical symmetry around the beam axis, the reaction cross section depends only on the difference of the azimuthal angles $\Phi_3 - \Phi_4$. This constitutes a kinematic over-specification of the system which is useful in reducing background.

For a given set of angles of detection ($\Theta_3$, $\Phi_3$) and ($\Theta_4$, $\Phi_4$) the results are conveniently presented in an experimental space. Energy and momentum conservation restrict the kinematically allowed values of $T_3$ and $T_4$ to a geometrical locus given by the equation \textsuperscript{14})

$$Q = \left(1 + \frac{M_3}{M_5}\right) T_3 + \left(1 + \frac{M_4}{M_5}\right) T_4 - \left(1 - \frac{M_1}{M_5}\right) T_1 - 2 \cos \Theta_3 \left(\frac{M_1}{M_5} T_1 T_3\right)^{\frac{1}{2}} - 2 \cos \Theta_4 \left(\frac{M_1}{M_5} T_1 T_4\right)^{\frac{1}{2}} + 2 \cos \Delta_{34} \left(\frac{M_3}{M_5} T_3 T_4\right)^{\frac{1}{2}},$$

where

$$\cos \Delta_{34} = \cos \Theta_3 \cos \Theta_4 + \sin \Theta_3 \sin \Theta_4 \cos (\Phi_3 - \Phi_4)$$

and $\Theta_i$, $\Phi_i$, $M_i$ and $T_i$ are the polar angles, azimuthal angles, mass and kinetic energy of particle $i$, and $Q$ is the $Q$-value of the reaction. Examples of kinematic loci are given in fig. 2.
As one moves on the $T_3, T_4$ plane along the kinematic line for a three-body reaction, the difference in arrival time $\Delta t$ of the two detected particles varies continuously. Under the experimental conditions employed here this parameter covers a range of a few nanoseconds. In this case, a coincidence circuit with a resolving time less than a nanosecond can set a time window, which will include only a segment of the kinematic line. The procedure used in the present experiment was as follows: the TAC output, whose pulse-height is proportional to the parameter $\Delta t$, was digitized into six channels by the "TIME" ADC. As described in the previous section, the data-taking program stores the coincident events in six $64 \times 64$ channel arrays according to the "TIME" ADC output. Thus the segment of the kinematic line satisfying the $\Delta t$ requirement set by each time channel appears in the corresponding energy array.

![Diagram](image)

Fig. 2. Kinematics of the reactions occurring with a $^3\text{He}$ beam incident on CD$_2$ target. $T_1 = 24.0$ MeV, $\Theta_x = 20^\circ$, $\Theta_y = -50^\circ$. The two large closed curves arise from eqs. (2a, b). The curved line along the diagonal is due to the final state in eq. (2b) with the two protons detected. The small closed curve is due to eq. (2c) and the almost straight line on the outside due to eq. (3b).

The above data-taking scheme may be conveniently employed for particle identification. In a three-body reaction with three distinct charged particles in the final state, one in general has to consider a maximum of six kinematic lines on the $T_3, T_4$ plane, characterized by the particular pair of particles detected. The number of kinematic lines increases when competing channels are open. The reaction $^3\text{He} + ^2\text{H}$, for example, will give rise to eight kinematic lines, due to reactions (2). Since a CD$_2$ target was used in this experiment, the spectrum will be further complicated by the appearance of lines arising from the reactions

\[ ^3\text{He} + ^{12}\text{C} \rightarrow ^2\text{H} + p + ^{12}\text{C}, \quad (3a) \]

\[ ^3\text{He} + ^{12}\text{C} \rightarrow p + p + ^{13}\text{C}. \quad (3b) \]
For a given set of detection angles and incident $^3$He energy the situation is shown in fig. 2. It is seen that in the experimental spectrum some of the kinematic lines overlap and certain channels of the $T_3$, $T_4$ array will contain events contributed to by more than one final state of the reaction. This ambiguity is often eliminated if one takes into account the time-energy correlation of the events. Through judicious selection of detector distances from the target, the difference in $\Delta t$ for two different types of events which overlap in energy usually can be made greater than the coincidence circuit resolving time. In such a case overlapping segments of kinematic lines will be stored in different time-areas and particle identification is effected. The situation is exemplified in fig. 3, which represents typical spectra stored in the on-line computer during the data-taking mode: The last spectrum is obtained by imposing a broad coincidence requirement. The long diagonal line in this spectrum, corresponding to the detection of the two protons in reaction (2b), is clearly separated in time-channels 3 and 4 and may be conveniently summed after the completion of the experiment.

3. Experimental results and analysis

3.1. p-n FINAL STATE INTERACTION IN THE REACTION $^4$He+$^2$H

Reaction (1) was studied at 24.0 MeV incident $^4$He energy and the $^4$He and proton in the final state were detected at angles $\Theta_3 = 22^\circ$ and $\Theta_4 = 42^\circ$ with respect to the incident beam, respectively; $\Theta_4$ was selected to be greater than the maximum angle in which $^4$He is kinematically allowed to scatter so that the opposite detection scheme is not allowed. This simplifies considerably the spectrum obtained.

The experimental spectrum is shown in fig. 4. The energy scale for the protons is 0-10 MeV and for $^4$He is 7-15 MeV with the direction of increasing energy along the axes as indicated in the figure. The large peak at channel $^*$ (52, 23) is identified from the kinematics as due to the ground state of $^5$He, formed by the sequential process $^4$He+$^2$H $\rightarrow$ p+$^5$He(g.s.) $\rightarrow$ p+$^4$He+n.

A simulation of the data – not shown in the figure – assuming a Breit-Wigner shape and acceptable values for the position and width of this state relative to the n-$^4$He system ($E^* = 0.957$ MeV, $\Gamma = 0.58$ MeV), was able to reproduce the shape of the peak. This peak has been analyzed in more detail elsewhere $^{13,14}$).

The two-nucleon enhancement factors that describe the three-particle cross section are given in ref. $^8$). The p-n interactions for both spin states attain a strength maximum at zero relative energy. The triplet state interaction has a smooth variation with relative energy and follows approximately the structure of phase space. On the other hand, the singlet interaction is expected to exhibit a resonance-like behavior at zero p-n relative energy. From the kinematics, the proton and neutron in the final state

$^*$ We will refer to channels according to the convention $(y, x)$ with the coordinate system defined in fig. 2. We also note that the channel grid of the three-dimensional display is illuminated with twice the ordinary intensity every eighth channel.
of eq. (1) are emitted with zero relative energy for laboratory kinetic energies near channel (24, 50). There is, however, no apparent resonance near this channel in the experimental spectrum.

Two simulations of the data are given in fig. 4 by assuming FSI in the singlet and triplet states, independently. The calculation of the theoretical spectra is explained in ref. 8. In this case the three-body cross section that enters into the simulation program is given by

$$\frac{d^4\sigma}{dT_3dT_4d\Omega_3d\Omega_4} = N \times \Pi \times \text{FSI} \times \text{PS},$$

where \( N \) is an arbitrary normalization factor and \( \text{PS} \) is the expression for phase space. \( \Pi \) is a factor which describes the primary interaction and, for the calculations presented in this section, has been taken equal to 1. The factor \( \text{FSI} \) describes the two-nucleon final state interaction and, for the two simulations in fig. 4, has been taken equal to the appropriate enhancement factor \( 8,16 \).

$$\text{FSI} = \left( \frac{1}{r_0} + \frac{1}{a} + \frac{1}{2}r_0k^2 \right)^2, \quad \frac{k^2}{r_0} + \left( \frac{1}{a} + \frac{1}{2}r_0k^2 \right)^2,$$

where \( r_0 \) is the effective range and \( a \) the scattering length. In general, the proton and neutron will interact in both spin states and FSI takes the form

$$\text{FSI} = E^s_{pn} + T/S E^t_{pn},$$

where \( T/S \), the triplet-to-singlet ratio, expresses the relative probability for the \( p-n \) system to emerge from the primary interaction in a spin \( I = 1 \) state and \( E^s_{pn} \) and \( E^t_{pn} \) are the enhancement factors for the two spin states. For calculational purposes, this is equivalent to adding the two theoretical spectra in fig. 4 weighted according to \( T/S \).

The accepted values of scattering lengths and effective ranges for the two spin states were used \( 13 \)

$$a_s = -23.68, \quad r_s = 2.5, \quad a_t = 5.40, \quad r_t = 1.7 \quad \text{(fm)}.$$  

To facilitate the presentation of the comparison of theory and experiment, the events contained in the rectangular area between channels 1 and 64 along the \( y \)-axis and channels 40 and 64 along the \( x \)-axis, were projected onto the \( y \)-axis. This rectangular section includes the segment of the kinematic line, which is of interest in the analysis. Fig. 5 contains the projection of the data together with projections of the simulations for two values of \( T/S \). The actual comparison of theory and experiment is performed by evaluating the expressions

$$\chi^2 = 1 \sum_{i=1}^{N} \frac{(x^\text{sim}_i - x^\text{exp}_i)^2}{x^\text{exp}_i},$$

where...

...
where $x_i^{\text{sim}}$ and $x_i^{\text{exp}}$ are the counts in the $i$th channel of the theoretical and experimental spectrum, respectively. If it is assumed that the only difference between the experimental and simulated data is statistical, then it can be shown that $\chi^2$ will converge to 1 for a large number ($N$) of densely populated channels. Theoretical spectra were constructed by adding the triplet and singlet simulations for a range of $T/S$ values. The results of the $\chi^2$ test are shown in fig. 6. Each of the curves in the figure corresponds to a particular range of channels about the singlet peak considered in the comparison. It is observed that the minimum of all curves occurs at $S/T = 0$; that is, the data is best reproduced by considering the p-n interaction in the triplet state only. This, of course, is the prediction of isospin conservation. An estimate of the uncertainties in this method of analysis yields an upper limit of 3% singlet state admixture.

### 3.2 Knock-out Poles in the Reaction $^3\text{He} + {^2}\text{H}$

Reaction (2a) was studied at 24.0 MeV $^3\text{He}$ incident energy at angles which enhance quasi-free scattering $^{16,18}$. These are angles for which the reaction

$$^3\text{He} + p \to ^3\text{He} + p, \quad Q = -2.226 \text{ MeV},$$

(7)
is realizable. The meaning of eq. (7) is that the incoming $^3\text{He}$ interacts only with the bound proton in the deuteron. By assuming a Hulthén-type wave function for the deuteron, the amplitude for this process is given in the plane-wave-Born approximation by $^{18}$

$$(\psi_f|V|\psi_i) \propto \left( \frac{\alpha}{\alpha^2 + k_n^2} - \frac{\beta}{\beta^2 + k_n^2} \right), \quad (8)$$

where $h\mathbf{k}_n$ is the momentum of the neutron in the final state and $\alpha = (mW/h^2)^{1/2}$, $\beta = 7\alpha$; $W$ is the binding energy of the deuteron and $m$ its mass.

Eq. (8) predicts an intensity maximum at zero laboratory neutron momentum. Spectator peaks have been observed in many reactions, but the maxima rarely coincide with the position and width in the spectra predicted by the simple Born approximation $^{19,20}$. A theoretical fit to the experimental spectra is usually obtained by either taking an artificially smaller binding energy or shifting the maxima to positions corresponding to some nonzero value of the momentum transfer $^{20}$. These effects are usually attributed to the long-range Coulomb interaction, and the analysis of the present experiment attempts to take into account Coulomb effects through a simple one-parameter model. The assumption of this model is that before the short-range nuclear interaction takes over, the Coulomb field of the incoming $^3\text{He}$ particle transfers to the deuteron an average momentum $\mathbf{p}_D$. In this case eq. (8) takes the form

$$(\psi_f|V|\psi_i) \propto \left( \frac{\alpha}{\alpha^2 + q^2} - \frac{\beta}{\beta^2 + q^2} \right), \quad (9)$$

where now

$$q = \frac{1}{2}k_D - k_n$$

and $\frac{1}{2}h\mathbf{k}_D$ is half the average momentum transferred to the deuteron before the nuclear collision and may be used as a parameter to be fitted for the best agreement between theory and experiment. The experiment was conducted at $\Theta_3 = 14^\circ$ and $\Theta_4 = -42^\circ$ for the detection of $^3\text{He}$ and protons, respectively. The maximum angle at which $^3\text{He}$ can be emitted is $36^\circ$, so that, as in the previous section, the opposite detection possibility is not realizable and only one kinematic line will appear. However, the experimental spectrum is now complicated by the appearance of events due to the competing channel

$$^3\text{He} + ^2\text{H} \rightarrow ^3\text{H} + \text{p} + \text{p}, \quad Q = -1.462 \text{ MeV.}$$

Since the reactions (2a) and (2b) have comparable $Q$-values, their kinematic lines are close in energy. Furthermore, since tritons and protons have the same mass, the only substantial difference in time-energy correlation of ($^3\text{He}$, p) and ($^3\text{H}$, p) coincident events is due to the difference in charge-collection time in the detectors for $^3\text{He}$'s and tritons. With the present resolving time of the apparatus this difference is too small to effect a separation of the two kinematic lines into time-areas. Thus the separation of the two kinematic lines was not complete, and a small overlap remained.
In fig. 7 the three-body spectrum obtained in the experiment is given, with the direction of increasing energy along the axes indicated in the figure. The energy scales are 0-20 MeV and 0-16 MeV for \(^3\)He's and protons, respectively. In the same figure two full simulations of the experiment are shown. The theoretical cross section is again given by eq. (4). Since in this case we are not near any final state interaction resonance, we may approximate

\[ \text{FSI} = 1 \]

near the area of interest; that is, no marked deviations from phase space due to final-state interaction are expected. Strong modulations in the three-body cross section will be due to the primary interaction term PI. When both primary and final state interaction are present, PI may be approximated by \(^8\)

\[ \text{PI} = (1 - C_s) + C_s \text{ QFS}, \tag{10} \]

where \(C_s\) describes the strength of the spectator effect and QFS is the square of the transition matrix element in eq. (9). In the previous section we approximated eq. (10) by setting \(C_s = 0\). Since the detection apparatus parameters were chosen in order to enhance the opposite extreme, the appropriate approximation here is

\[ C_s = 1. \]

The term QFS in the form employed by the simulation program, is written from eq. (9) as

\[ \text{QFS} = \left[ \frac{1}{BE + TW} - \frac{1}{CE + TW} \right]^2, \tag{11} \]

where

\[ TW = \frac{T_{DC}}{2} + T_5 - (2T_{DC}T_5)^{\frac{1}{2}} \cos \Theta_5. \tag{12} \]

Above, BE is one-half the binding energy of the deuteron \(BE = 1.113\) MeV, \(CE = 49\) BE and \(T_{DC}\) is the average energy (in MeV) transferred to the deuteron before the nuclear interaction takes place. The two simulations in fig. 7 are generated for two values of the parameter \(T_{DC}\) which is to be fitted.

If the comparison of theory to experiment is performed by assuming a stationary target deuteron, the result is a discrepancy in the centroid of the projected peak. Nine simulations were performed by varying the parameter \(T_{DC}\) in eq. (12) from 0-160 keV in 20 keV steps.

From both the centroid position and a \(\chi^2\) fit it is found that the best agreement is obtained for 130 keV average deuteron "Coulomb" kinetic energy. This corresponds to a spectator momentum transfer of 11 MeV/c or recoil energy of the spectator neutron of 65 keV. If we assume that in a head-on collision the incoming \(^3\)He transfers "Coulomb" energy to the deuteron up until the nuclear interaction occurs, then this
interaction distance is about 1.1 fm. Considering the simplicity of the model this is a reasonable result.

3.3. p-n FINAL STATE INTERACTION IN THE REACTION $^3\text{He}+^2\text{H}$

The interaction of the proton and undetected neutron in the final state of reaction (2a) was studied at 18.0 MeV and 24.0 MeV incident $^3\text{He}$ energy. In order to study the fine details of this interaction, the energy axes of the three-dimensional spectra were expanded so that the p-n FSI peak covered a large number of channels. The experimental parameters in both measurements are given below. The scale of the energy is defined by giving the energy of channels 16 and 64.

<table>
<thead>
<tr>
<th>$^3\text{He}$ lab. energy (MeV)</th>
<th>18.0</th>
<th>24.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_x (^3\text{He})$</td>
<td>$-30^\circ$</td>
<td>$-30^\circ$</td>
</tr>
<tr>
<td>$\Theta_y$ (protons)</td>
<td>$37.9^\circ$</td>
<td>$41.7^\circ$</td>
</tr>
<tr>
<td>x-axis, channel 64 (MeV)</td>
<td>10.5</td>
<td>13.0</td>
</tr>
<tr>
<td>x-axis, channel 16 (MeV)</td>
<td>6.5</td>
<td>9.0</td>
</tr>
<tr>
<td>y-axis, channel 64 (MeV)</td>
<td>8.0</td>
<td>12.0</td>
</tr>
<tr>
<td>y-axis, channel 16 (MeV)</td>
<td>2.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The experimental spectra thus obtained are shown in figs. 8 and 9. The 18.0 MeV data were accumulated in approximately 5 hours running time by maintaining a 30 nA $^3\text{He}$ beam on a 0.5 mg/cm$^2$ CD$_2$ target. The experimental spectrum shown in fig. 8 contains about 200 counts per channel near the peak maximum. The running time for the accumulation of the 24.0 MeV data was approximately 14 hours with a $^3\text{He}$ beam of 10 nA. The $T_3, T_4$ scale was expanded for the 24.0 MeV run in order to spread the final state interaction peak over more channels. The 24.0 MeV peak is sharper than the one at 18.0 MeV. This arises because the tail of the $^3\text{He}$-n resonance contributes significantly more strongly to the background under the 18.0 MeV peak in accordance with the lower c.m. energy available in this case: 4.97 MeV vs 7.37 MeV for the 24.0 MeV data. This degradation in sharpness at lower c.m. energy results in a decrease in sensitivity to the scattering length $a_s$, as has been observed in other reactions.

Simulations of the data were generated on the SDS Sigma 5 computer at Rutgers. In reaction (2a) the p-n final state interaction is allowed in either spin state, so that the expression for the three-body cross section takes the form of eq. (4) with the term FSI given by eq. (5). In comparing theoretical to experimental spectra, the singlet scattering length, the ratio $T/S$ and the effective target thickness were treated as parameters. Assuming a singlet effective range $r_s = 2.6$ fm and varying the scattering length between $-18.0$ and $-28.0$ fm, in two-fermi steps, six simulations for each set of data were computed. The nonresonant interaction in the triplet state was simulated by assuming $r_t = 1.7$ fm and $a_t = 5.40$ fm. The theoretical spectra were corrected for target thickness by matching the spread of the kinematic locus and the peak maximum position in the data with the help of the SDS-925 computer display.
Similarly, the triplet-to-singlet ratio was determined by reproducing as nearly as possible the height of the "wings" of the peak in the experimental spectra. The values $T/S = 5.0$ and $T/S = 6.0$ were obtained in this way for the 18.0 MeV and 24.0 MeV data respectively. These values are not accurate representations of the true triplet-to-singlet ratio since the sloping tail due to the $^3$He-\(n\) resonance was not included in the calculation. This tail is evident in fig. 9.

Theory was compared to experiment by evaluating the $\chi^2$ function of eq. (6) for each of the six scattering lengths. The $\chi^2$ values were fit to parabolae and the minima of these parabolae were found to cluster about $-24.0$ fm with a spread of $\pm 4$ fm depending on which region of the peak was included in the $\chi^2$ computation. This inconsistency in the best value of the scattering length is explained by a slight mismatch in shape between the simulation and the experiment, primarily due to the neglect of the $^3$He-\(n\) resonance, and partly due to uncertainty in the correct values of the adjustable simulation parameters since there were insufficient statistics to determine these as accurately as is necessary.

3.4. EXCITED STATES OF $^4$He

The initial state $^3$He + $^2$H may proceed to the final state of two protons and a triton sequentially through excited states of $^4$He, i.e.

\[
^3\text{He} + ^2\text{H} \rightarrow ^4\text{He}^* + p
\]

\[
\Downarrow \rightarrow ^3\text{H} + p.
\]  

(13)

This reaction was studied at 24.0 MeV incident $^3$He energy by detecting in coincidence the triton and one of the protons in the final state for a fixed proton angle $\Theta_x = -50^\circ$ and three triton angles $\Theta_y = 20^\circ$, 22$^\circ$ and 24$^\circ$. The experimental spectra are shown in fig. 10. The triton energy scale is from 0 to 20 MeV and the proton energy scale from 0 to 12 MeV.

In the overall c.m. system (SCM) the proton and $^4$He are emitted in opposite directions (fig. 11) and since the proton laboratory angle of detection is kept fixed in all spectra, it follows that the SCM angle of emission for the formation of a certain excited state of $^4$He is also fixed. This ensures the same primary production mechanism for a given excited state of $^4$He in all spectra. The $^4$He* will decay into a triton and a proton with a relative velocity $V_4 + V_5$ of the two decay products in their own c.m. system (RCM) determined by the difference in energy between the $^4$He excited state and the $^3$He+p threshold. Thus two RCM angles of emission of the triton (one being $\Theta_4$) are defined by the intersection of a circle with radius equal to the magnitude of the triton RCM velocity (small circle in fig. 11) and the direction with respect to the incident $^3$He beam, as defined by the triton laboratory angle of detection $\Theta_4$. The two RCM angles lead to two contributions for any particular $^4$He excited state along the $T_3$, $T_4$ kinematic line.
The angular correlation from a sequential process, proceeding through excited states which decay into two fermions, has been worked out by Zupančič \(^{18,21}\). For an isolated excited state with spin \(J\) and parity \(P\) the decay pattern has the form

\[
W(\theta, \phi) = \sum_{l=0}^{2J} \sum_{m=0}^{l} \text{Re} \left[ A_{lm}^{(J, P)} Y_{lm}(\theta, \phi) \right],
\]

where \(A_{lm}^{(J, P)}\) are complex coefficients depending on the excitation process and only even \(l\)'s and \(m\)'s occur in the sum. Thus for \(J = 0\) the pattern is isotropic and for \(J = 1\) and \(J = 2\) takes the following form in the case of coplanar events

\[
\begin{align*}
J = 1; & \quad W(\theta) = 1 + A \sin^2 \theta, \\
J = 2; & \quad W(\theta) = 1 + A \sin^2 \theta + B \sin^4 \theta,
\end{align*}
\]

where \(A\) and \(B\) are real coefficients.

![Diagram](image)

Fig. 11. The c.m. System (SCM) and Recoil c.m. System (RCM) for a three-body reaction. Capital letters denote laboratory variables; primed capital letters refer to the overall c.m. System (SCM) and small letters to the Recoil c.m. System (RCM). \(V_{\text{c.m.}}\) is the c.m. velocity in the initial state.

The triton RCM angles are the angles in the arguments of the spherical harmonics \(Y_{lm}\) in eq. (14), which describe the decay pattern of the excited \(^4\text{He}\) state. In order to investigate this angular pattern, the triton laboratory angle \(\Theta_4\) was varied in such a way as to explore most of the RCM circle (fig. 11) of the first excited state of \(^4\text{He}\). For the laboratory angles \(\Theta_3\) (proton) and \(\Theta_4\) (triton) the situation is shown in table 1 (we denote SCM angles by primed capital letters and RCM angles by lower case letters).

The experimental spectra were simulated by assuming no structure due to the primary interaction (\(C_s = 0\)) and a sum of Breit-Wigner resonances for the term FSI.
of eq. (4) in the three-body cross section

\[
\text{FSI} = \sum_i C_i \frac{\exp - G_a}{(E_{\text{res}} - E_{\text{rel}})^2 + \frac{1}{4} \Gamma^2},
\]

where \( G_a \) is given by \(^{22}\)

\[
G_a = \frac{\pi Z_i Z_j}{\hbar |V_i - V_j|};
\]

\( \exp - G_a \) approximates the Coulomb penetration function \(^{16}\). The symbols \( V_i \) and \( Z_i \) are the velocity and charge of the interacting particles and \( C_i \) are parameters describing the probability that the primary interaction leaves the interacting pair in a particular \(^4\)He excited state. The sum in eq. (15) includes all the excited states of \(^4\)He through which the reaction is considered to proceed to the final state. It should be noted that eq. (15) is valid only in the case of well-separated resonances. As resonances begin to overlap, interference terms between different angular momenta may become important \(^{23}\). In the case of excited states of \(^4\)He, eq. (15) is, therefore, expected to be valid for the description of the first excited state and possibly the shape of the low energy slope of the second excited state resonance.

### Table 1

<table>
<thead>
<tr>
<th>( \theta_3 ) (p)</th>
<th>( \theta_4 ) ((^3)H)</th>
<th>( T_3 ) (MeV)</th>
<th>( T_4 ) (MeV)</th>
<th>( \theta'_3 ) (p)</th>
<th>( \theta'_4 ) ((^3)H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50° 20°</td>
<td>10.565</td>
<td>7.611</td>
<td>81.1°</td>
<td>188.5°</td>
<td></td>
</tr>
<tr>
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<td>10.189</td>
<td>81.1°</td>
<td>100.9°</td>
<td></td>
<td></td>
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<tr>
<td>50° 22°</td>
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<td>7.579</td>
<td>81.1°</td>
<td>93.5°</td>
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<td>67.1°</td>
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<td></td>
</tr>
<tr>
<td>50° 24°</td>
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<td>7.823</td>
<td>81.1°</td>
<td>65.3°</td>
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</tr>
<tr>
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<td>9.864</td>
<td>81.1°</td>
<td>35.3°</td>
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</tbody>
</table>

The two peaks in the spectra of fig. 10, which are due to the first excited state of \(^4\)He (near the highest proton energy) are found to be at points along the kinematic line which correspond to a relative energy between the triton and the undetected proton of about 200 keV. The best fit of these peaks in all spectra was obtained for \( E_{\text{res}} = 200 \pm 20 \) keV, \( \Gamma = 120 \pm 20 \) keV. This corresponds to an excited state in \(^4\)He at \( E = 20.01 \pm 0.02 \) MeV. The large broad peak at higher triton energies in each spectrum is found to be centered at a relative energy of the triton and the undetected proton of 1.55 \( \pm \) 0.1 MeV. This corresponds to a state in \(^4\)He at 21.36 \( \pm \) 0.1 MeV. If a width \( \Gamma = 2.0 \pm 0.2 \) is assumed for this state, the shape of the left side of the peak is fairly well reproduced. The comparison of theory and experiment was performed visually on the display of the Bell-Rutgers SDS-925 computer and the errors quoted
here in the position and width of the states represent the smallest interval in the parameters that produced visible differences between theory and experiment.

A simulation of the $\theta_x = -50^\circ, \theta_y = 22^\circ$ data, assuming that only these two states contribute, is shown in fig. 12, together with the corresponding experimental spectrum. It is seen that the long ramp on the right side of the large peak which is due to the 21.36 MeV state is not reproduced in the theoretical spectrum. Similarly the second peak due to this state, expected around channel (18.47) is far too small in the simulation. This situation is qualitatively explained by assuming contributions from higher states of $^4$He. As one moves away from the highest proton energies along the kinematic line, the triton-proton relative energy increases and resonances from higher states dominate in eq. (15). In addition interference terms probably become important and the simple description given by eq. (15) is not applicable in this region.

Fig. 13. Angular distribution of tritons from the decay of the first excited state of $^4$He.

Since the second $^4$He excited state is so broad that it overlaps higher states, angular correlation analysis was performed for the first excited state only. The elastically scattered deuterons at 50° were monitored during the runs for each spectrum (approximately $6 \times 10^6$ deuterons for each set of angles) and were used for normalization. As the simulation program inherently assumes an isotropic decay in the RCM system, the normalization constants $N$ in eq. (4) needed for the normalization of the three simulations to the experimental spectra were compared. Each spectrum contains two contributions from the first excited state of $^4$He, corresponding to two different RCM angles of emission for the triton. Thus isotropy in the $^4$He* decay pattern requires that the relative height of the two experimental peaks in each spectrum be
reproduced by the simulation. This was found to be the case, within a few per cent, in all spectra. The angular correlation data are shown in fig. 13.

The approximate isotropy in this decay pattern is consistent with the assignment of zero spin to the first excited state of $^4$He. The parity, however, of the state is not determined by this experiment. A zero spin state may be formed by a singlet triton-proton interaction in orbital angular momentum $L = 0$ ($J^\pi = 0^+$) or by a triplet interaction with $L = 1$ ($J^\pi = 0^-$).

4. Discussion and conclusions

The measurement of seven scalar parameters in the final state of the three-body reaction by coincident methods, enables individual features of these reactions to be separated from competing processes. This separation could be further improved if higher c.m. energies could be obtained. If particle identification is not available, a difficulty occurs due to the closeness of kinematic lines involving particles of equal mass and having comparable $Q$-values, such as the $^3$H+p+p and $^3$He+p+n final states.

Our simple final state interaction theory, which is based on the Watson model 24), produces reasonable fits to the proton-neutron final-state interaction peak in the $^3$He+$^2$H reaction. The uncertainty in the extraction of the singlet scattering length of $\pm 4$ fm could be reduced by obtaining higher statistics, and running at higher energy. In the final state of the reaction $^4$He+$^2$H $\rightarrow$ $^4$He+p+n the p-n FSI in the singlet state was found to be absent in accordance with the prediction of isotopic spin conservation. The upper limit of 3 % in $S/T$ quoted in the text should be viewed as a qualitative indication of isospin conservation in this reaction. A quantitative measurement of isospin conservation would require a knowledge of the value of $S/T$ to be expected if isospin were completely broken.

The simple, one-parameter, “Coulomb pushing” model of quasi-free scattering reproduced the observed shift in spectator peaks. It should be interesting to apply this model to experimental data containing larger deviations from the plane wave Born approximation predictions, as those of K. Bähr et al. 20).

The Breit-Wigner resonance was found to produce adequate fits for the extraction of spectroscopic data from intermediate-state excited states of $^4$He in regions along the kinematic line, where no significant overlap of states was present. This was particularly true for the first excited state of $^4$He for which angular correlation measurements were also performed. The angular correlation of the decay particles from this state was found to be consistent with the $J = 0$ assignment which has been previously reported 2). Some of the position and width parameters of the first and second excited states of $^4$He as seen in this experiment differ significantly from those found both in the same reaction at higher $^3$He energy 6) and in other measurements such as proton-triton elastic scattering 2). Such phenomena can probably be explained by the effects that the three-body nature of the final state has on the reaction mechanism at different c.m. energies and other measuring conditions.
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